Collective motions of particles with diffusive interactions

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Reaction-diffusion systems can generate traveling spot solutions with a constant velocity. In the previous work [1], the interaction of spots is classified, and the explicit forms to describe the dynamics are obtained rigorously. Actually, some of traveling spot solutions exhibit spatiotemporal collective motions under the bifurcation structure with Jordan block type degeneracy. In this talk, we are concerned with the dynamics of spots near the bifurcation point. Using center manifold theory, we will reduce a reaction-diffusion system to an ordinary differential equation that describes positions and velocities of spots.

We also apply our result to the following system which is derived by modifying a model in [2] with a periodic boundary condition;

$$\begin{cases} x_i'' = \dfrac{\gamma}{2r}\left(\dfrac{1}{1+au(x_i+r,t)} - \dfrac{1}{1+au(x_i-r,t)}\right) - \mu x_i', & t>0, \\ \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} - ku + \displaystyle\sum_{i=1}^{n} F(x,x_i,r), & 0<x<L,\ t>0. \end{cases} \tag{P}$$

This model represents the self-sustaining motion of camphor disks in an annular water channel, where $x_i = x_i(t)$ and $u = u(x,t)$ denote the position of the i-th camphor disk ($i = 0,\dots,n$) and the surface concentration of the camphor layer under periodic boundary conditions, respectively. The constants γ, a, μ, k, r are positive, and the function $F(x,x_0,r)$ is defined by

$$F(x,x_0,r) = \begin{cases} \rho, & -r < x-x_0 < 0, \\ 1, & 0 < x-x_0 < r, \\ 0, & |x-x_0| > r \end{cases}$$

for a positive constant $0 < \rho < 1$. We will characterize several types of solutions in a reduced system through a bifurcation analysis.

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References

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