RESISTIVITY TENSOR IMAGING VIA NETWORK DISCRETIZATION OF FARADAY’S LAW*

MIN-SU KO† AND YONG-JUNG KIM‡

Abstract. A resistive network based anisotropic resistivity reconstruction method is introduced when three sets of internal electrical currents are given. Faraday’s law is discretized using resistive networks for an anisotropic resistivity body such as a muscle or nerve fiber structure of an animal body. The resistivity tensor imaging method of this paper may provide another approach to find the fiber structure of white matter. We take three sets of internal electrical current density and a part of boundary conductivity to construct the anisotropic conductivity in two space dimensions. The construction algorithm is explicit and local. Hence, the computation time is only of order of the space dimensions.

Key words. inverse problem, resistive network, tensor imaging, anisotropic conductivity

AMS subject classifications.

1. Introduction. A resistivity (or conductivity) body with cell membrane, muscle fiber, or nerve fiber structure must be investigated using an anisotropic tensor model (see [4, 28, 32]). The purpose of this paper is to develop a network based numerical method for resistivity tensor imaging. The introduced algorithm requires local computations only and hence the computation complexity is same as the order of space dimensions. This simplicity is important in achieving a fast reconstruction method for three space dimensions.

Let \( \Omega \subset \mathbb{R}^2 \) be a bounded simply connected domain of an anisotropic resistivity body and its boundary \( \partial \Omega \) be smooth. Anisotropic resistivity tensor \( \mathbf{r} = \mathbf{r}(\mathbf{x}) \) is a symmetric matrix,

\[
\mathbf{r} := \begin{pmatrix} r^{11} & r^{12} \\ r^{21} & r^{22} \end{pmatrix}, \quad r^{12} = r^{21}, \quad \mathbf{x} := (x, y) \in \Omega \subset \mathbb{R}^2.
\]

Let \( \mathbf{J}_k = (J^1_k, J^2_k) : \Omega \to \mathbb{R}^2, \ k = 1, 2, 3, \) be three given vector fields in the domain \( \Omega \). If these vector fields are electrical current densities through the resistivity body, then three sets of Faraday’s laws,

\[
\nabla \times (\mathbf{r}\mathbf{J}_k) = 0 \quad \text{in} \quad \Omega, \ k = 1, 2, 3,
\]

\[
\mathbf{r} = \mathbf{r}_0 \quad \text{on} \quad \Gamma \subset \partial \Omega,
\]

are satisfied. If current densities are given, these current equations provide relations between currents and the resistivity tensor. The uniqueness and existence of an anisotropic resistivity tensor that satisfies the system for the three given data \( \mathbf{J}_k, \ k = 1, 2, 3, \) have been shown when the current densities are admissible (see [18, Definition 3.1 and Theorem 3.11]). The uniqueness and existence theorems imply that three sets of current densities are the correct amount of data to obtain anisotropic resistivity.

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†Department of Mathematical Sciences, KAIST, 291 Daehak-ro, Yuseong, Daejeon 305-701, Korea (minsu.ko@kaist.ac.kr).

‡National Institute of Mathematical Sciences, 70 Yuseong-daero, Yuseong-gu, Daejeon 305-811, Korea AND Department of Mathematical Sciences, KAIST, 291 Daehak-ro, Yuseong, Daejeon 305-701, Korea (yongkim@kaist.edu, http://amath.kaist.ac.kr/pdeJab/).
tensor in a unique way. If more equations are added or subtracted, then the existence or the uniqueness will fail and the solution should be understood in a different sense.

A mimetic method is used in the numerical scheme of this paper. In a usual finite difference scheme, solution values at grid points are approximated. However, we consider a network system given as in Figure 1 and the resistivity values are assigned along edges. The same resistivity tensor is assigned to the edges in a square plate, where the plate and a network cell is slightly dislocated as pictured in Figure 1(b). For example, the two edges in the square plate, b and c, have the same resistivity tensor. Then a loop integral over a cell boundary of the network gives an equation for each current densities. (A detailed discussion is given in Section 2.) In return we obtain three equations for each cell which end up a linear system (3.4). However, to obtain a successful numerical algorithm, we should handle the singularity in the coefficient matrix and noise propagation along characteristic lines. The main part of this paper is a process to resolve these two issues which is given in Sections 3 and 4. Further numerical simulations are given in Section 5 to test the property of the scheme.

The network system in this paper is different from electrical resistive networks for two reasons. First, it is simply used as a discretization method of a continuum body. Second, more importantly, we need the component of the current which is normal to edge direction. Remember that the physical resistive network system requires currents only along each edge. Hence, we call the system virtual resistive network (VRN for brevity) and has been applied for isotropic and orthotropic cases (see [19, 20, 21]).

Resistivity (or conductivity) reconstruction techniques using internal current densities have been studied by many authors. The magnetic resonance current density imaging (MRCDI) technique is used for the internal measurement of current densities (see [12, 15, 35]). A MR measurement gives a single component of the magnetic field, say $B_z$, and the radial one $\sqrt{B_x^2 + B_y^2}$. Having a $B_z$ component is enough to find the current density in two space dimensions. However, to obtain all three components for three dimensional case, the subject should be rotated. A recent technology makes a full measurement possible without a full rotation, but by tilting the subject and is being used in diffusion tensor imaging. In fact the anisotropic structure of brain nerve connections has been intensively studied using diffusion tensor imaging technology. The fiber structure of white matter gives anisotropic diffusion dominated in the direction of the fibers (see [11, 29, 37, 34]). The resistivity tensor imaging technique developed in this paper may provide another approach to find the fiber structure of white matter using the anisotropy property in resistivity.

The uniqueness of anisotropic resistivity distribution has been shown for many cases, [7, 8, 9, 13, 22, 23, 24]. Most of such uniqueness results are obtained from an overdetermined problems and hence existence is not expected\(^1\). For example, Monard and Ball [22] showed such a uniqueness when four sets of internal power densities are given\(^2\). Note that there are three unknowns, $r^{11}$, $r^{12}$ and $r^{22}$, and one current vector field gives one equation in two space dimensions.

Electrical impedance tomography (EIT for brevity) is to find the internal conductivity distribution from a boundary measurement of voltage and has a long history (see [3, 6, 39]). The conductivity $\sigma = r^{-1}$ is the inverse tensor of the resistivity one

\(^{1}\)We may say this since we now know that three sets of internal current data are just enough to give the existence and uniqueness together.

\(^{2}\)It is also mentioned that they were able to compute anisotropic conductivity numerically only with three sets of data.
and the voltage $u$ satisfies
\begin{align}
\nabla \cdot (\sigma \nabla u) &= 0 \quad \text{in } \Omega, \\
-\sigma \nabla u \cdot n &= g \quad \text{on } \partial \Omega,
\end{align}
where $n$ is the outward unit normal vector to the boundary $\partial \Omega$ and $g$ is the normal component of the boundary current density that satisfies $\int_{\partial \Omega} g ds = 0$. If voltage $u$ is a given datum, this voltage equation gives a relation between voltage and conductivity. The conductivity $\sigma$ is uniquely determined by a relation between the boundary voltage and current (see [25, 38]) for isotropic conductivity cases, but not anisotropic ones (see [10]). Therefore, using internal data is unavoidable to obtain anisotropic conductivity tensor.

The use of internal data has been started for isotropic cases (see [1, 30, 31]). The uniqueness was shown in various cases (see [14, 16, 17, 26, 27, 36]). However, if given data are current densities, the voltage equation (1.3) does not stand alone and Ohm’s law, $-\sigma \nabla u = J$, should be added to make a connection between electrical currents and conductivity. Such an approach makes the problem highly nonlinear and unstable even with internal data. Furthermore, the method is not extended to an anisotropic case. Recently, Bal et al. showed uniqueness of an anisotropic conductivity reconstruction method [7, 8, 9, 22, 23, 24] using both properties of (1.2) and (1.3). Basically, an over-determined system was constructed using $n + 2$ current densities with space dimensions $n = 2$ and 3 and the uniqueness of the problem is shown.

2. Network discretization of Faraday’s law. Let $\Omega \subset \mathbb{R}^2$ be a bounded simply connected domain of a conductivity body and its boundary $\partial \Omega$ be smooth. We consider a static electromagnetic field $E$ in the body. First, the electric field satisfies the static Maxwell-Faraday equation, which is $\nabla \times E = 0$. Therefore, there exists a potential (or voltage) $u$ that satisfies $E = -\nabla u$. Let $\sigma : \Omega \rightarrow \mathbb{R}^{2 \times 2}$ be an anisotropic conductivity tensor distribution of the body. The resistivity tensor distribution is its inverse, i.e., $r = \sigma^{-1}$. The current density $J$ satisfies Ohm’s law:
\begin{align}
J &= \sigma E = -\sigma \nabla u \quad \text{or} \quad E = r J = -\nabla u.
\end{align}

The continuity equation for the electrical current is written as
\begin{align}
\nabla \cdot (\sigma \nabla u) &= 0 \quad \text{in } \Omega, \\
-\sigma \nabla u \cdot n &= g \quad \text{on } \partial \Omega.
\end{align}
This elliptic equation serves as a model equation in conductivity reconstruction problems and has intensively studied analytically and numerically.

If the voltage $u$ is given, one may find the conductivity $\sigma$ by using this voltage equation. If the current density $J$ is given, we may construct the resistivity $r$ similarly by using current equations, or Faraday’s law,
\begin{align}
\nabla \times (r J_k) &= 0 \quad \text{in } \Omega, \quad k = 1, 2, 3, \\
r &= r_0 \quad \text{on } \Gamma \subset \partial \Omega.
\end{align}
The reconstruction method of this paper is based on this curl free equation since the current density $J$ is the given information. Furthermore, this curl free equation allows us an algorithm based on local computations, which drops the computational complexity considerably and makes three dimensional computation possible.

This curl free equation is relatively less studied in comparison with the divergence free equation. Numerical algorithms developed for the curl free equation are
also limited. One may simply apply finite difference schemes to discretize the system. However, such schemes are mostly based on conservation laws of wave propagation and we could not exploit the nature of the curl free equation from such approaches. Instead, we employ the intrinsic property of a curl free equation. The curl free equation (1.2) implies that the vector field \( \mathbf{rJ} \) is a potential vector field and, therefore, a line integral of \( \mathbf{rJ} \) over any closed loop is zero as long as there is no singularity inside the loop. The discretization method of this paper is based on this property of potential vector fields.

\[
\begin{align*}
0 &= \int_D \nabla \times (\mathbf{rJ}) \, dx = \oint_{\partial D} \mathbf{rJ}(z) \, dz, \\
\mathbf{rJ} &= \left( r_{11}^1 J_1^1 + r_{12}^1 J_2^1, r_{11}^2 J_1^2 + r_{12}^2 J_2^2 \right).
\end{align*}
\]

For simplicity, we assume the cell is of square shape and the length of each side is small. Then, this contour integration gives a second order approximation of the curl free equation,

\[
\begin{align*}
r_{a}^{11} J_{1,a}^{a} + r_{a}^{12} J_{2,a}^{a} + r_{b}^{21} J_{1,b}^{b} + r_{b}^{22} J_{2,b}^{b} - r_{c}^{11} J_{1,c}^{c} - r_{c}^{12} J_{2,c}^{c} - r_{d}^{21} J_{1,d}^{d} - r_{d}^{22} J_{2,d}^{d} = 0,
\end{align*}
\]

where \((J_{1,a}^{a}, J_{2,a}^{a}), (J_{1,b}^{b}, J_{2,b}^{b}), (J_{1,c}^{c}, J_{2,c}^{c})\) and \((J_{1,d}^{d}, J_{2,d}^{d})\) are electrical current vectors at the middle of corresponding edges. Suppose that the resistivity tensors at edges \(a\) and \(d\) are given by the boundary condition or from previous steps. Then, the equation is written as

\[
\begin{align*}
r_{a}^{21} J_{1,a}^{a} + r_{b}^{22} J_{2,b}^{b} - r_{c}^{11} J_{1,c}^{c} - r_{c}^{12} J_{2,c}^{c} &= g,
\end{align*}
\]

where the right side \(g\) is a given value by

\[
\begin{align*}
g := r_{a}^{21} J_{1,a}^{a} + r_{d}^{22} J_{2,d}^{d} - r_{a}^{11} J_{1,a}^{a} - r_{a}^{12} J_{2,a}^{a}.
\end{align*}
\]

The resistivity tensor at the corresponding cell consists of the four unknowns, i.e.,

\[
\begin{align*}
\mathbf{r} &= \left( \begin{array}{cc} r_{11} & r_{12} \\ r_{21} & r_{22} \end{array} \right).
\end{align*}
\]
Since the resistivity tensor is symmetric, we take \( r_{12}^c = r_{21}^b \). Then, (2.5) is written as
\[
\frac{r_{22}^b J_{2,b}}{J_{2,b}} + \frac{r_{12}^c (J_{1,b} - J_{2,c})}{J_{1,c}} - \frac{r_{11}^c J_{1,c}}{J_{1,c}} = g,
\]
which contains three unknowns. This is the fundamental relation for our resistivity reconstruction algorithm. Therefore, if there are three sets of current data, then we may construct the correct number of equations to solve the problem. Note that only the component of a current vector along each edge is needed in an ordinary resistive network system. However, we also need a normal component for the anisotropic case.

Orthotropic resistivity is a case when \( r_{12}^c = r_{21}^b = 0 \). Then, the relation is simplified to
\[
\frac{r_{22}^b J_{2,b}}{J_{2,b}} - \frac{r_{11}^c J_{1,c}}{J_{1,c}} = g, \quad g := \frac{r_{22}^d J_{2,d}}{J_{2,d}} - \frac{r_{11}^a J_{1,a}}{J_{1,a}}, \quad r_{12}^c = r_{21}^b = 0,
\]
which contains two unknowns. Furthermore, only the component of a current along each edge is used in this relation. In other words, a usual resistive network covers this orthotropic resistivity. The isotropic resistivity is a case that \( r_{11}^c = r_{22}^b \) and \( r_{12}^c = r_{21}^b = 0 \). Hence, we have
\[
r_{11}^c (J_{2,b} - J_{1,c}) = g, \quad g := r_{11}^d (J_{2,d} - J_{1,a}), \quad r_{12}^c = r_{21}^b = 0, \quad r_{11}^c = r_{22}^d,
\]
which contains one unknown. Therefore, an isotropic resistivity tensor can be constructed from a single set of electrical current density.

The main advantage of the previous algorithm is that the resistivity is obtained only from local computations. In other words, the computation complexity of the algorithm is same as the space dimension and we find a hope for three dimensional computation from this simplicity. However, the scheme has various properties which are not understood well. In the following sections we study the properties of the scheme and develop ways to improve it.

### 3. Noise propagation along characteristic lines

The existence of a resistivity tensor distribution that satisfies the curl free equation (2.3) has been proved by constructing it analytically using a characteristic method [18, 19, 20]. However, such a method cannot be implemented as a numerical method since noise is trapped along the characteristic lines and unwanted stripes are produced along characteristic lines (see [2, 5] for an alternative approach based on a viscosity-type regularization). Such a behavior appears from all of isotropic, orthotropic and anisotropic cases. We discuss such phenomena briefly and review techniques how to overcome them for isotropic and orthotropic cases in Sections 3.1 and 3.2. These techniques have been introduced in authors’ earlier papers [19, 20] and briefly discussed here again to compare the difference of the anisotropic case more clearly. Related obstacles for an anisotropic case is introduced in Section 3.3. The contribution of this paper is in developing numerical techniques to overcome the obstacles of anisotropic case which is done in Section 4.

#### 3.1. Isotropic case

Isotropic resistivity is given by a scalar tensor. Most of conductivity reconstruction methods have been developed for isotropic cases. Characteristic lines of an isotropic case are simply equipotential lines. In Figure 2 reconstructed resistivity images are compared using four different methods. All of these methods are obtained from a single set of current data. Note that electrical currents used in these examples flow in the direction from the left upper corner to the right bottom one and the tripees are exactly in the direction of characteristic lines. These noise strips are almost disappeared when a resistive network method is used, Figure
Fig. 2. Isotropic resistivity reconstruction images using a single set of current data with 10% multiplicative noise. (The figures and detailed discussions were published in [19].)

2(d). However, this is because cells are not aligned in the direction of characteristic lines and the issue still remains in other situations.

Next we consider issues of network methods. Using the previous boundary condition, the relation in (2.9) is written as

$$r_{11}^c = \frac{g}{J_{2,b}^c - J_{1,c}^c}.$$  

There are two critical issues. The first one is the singularity in the algorithm when $J_{2,b}^c - J_{1,c}^c \approx 0$. Therefore, the current direction was chosen in a way to reduce this singularity. The boundary conditions should be appropriately chosen depending on the current data which is given in the admissibility condition [19, Definition 2.1]. The quality of the fourth figure in Figure 2 was obtained since the singularity was avoided. The second issue is the noise propagation along the characteristic lines which can be observed from the first three images in Figure 2. If the characteristic lines are parallel to the alignment of network cells, the noise is trapped along the network cells and noise lines will appear even from the fourth images. However, the characteristic lines are in diagonal directions in the example and hence noise strips were minimized.

### 3.2. Orthotropic case

An orthotropic resistivity tensor is a diagonal matrix,

$$r(x) := \begin{pmatrix} r_{11}^c(x) & 0 \\ 0 & r_{22}^b(x) \end{pmatrix}, \quad x := (x, y) \in \Omega \subset R^2.$$  

In this case the reconstruction relation (2.8) contains two unknowns and hence two equations related to each cell are required. Suppose that we have two sets of current data $J_1$ and $J_2$. Then, we obtain two equations from each cell,

$$r_{b}^{22} J_{2,b}^2 - r_{c}^{11} J_{1,c}^1 = g_1,$$
$$r_{b}^{22} J_{2,b}^2 - r_{c}^{11} J_{1,c}^1 = g_2,$$

where $g_1$ and $g_2$ are given by relations in (2.8). Using matrix multiplication, it is written as

$$\begin{pmatrix} -J_{1,c}^1 & J_{2,b}^2 \\ -J_{2,c}^2 & J_{1,b}^1 \end{pmatrix} \begin{pmatrix} r_{11}^c \\ r_{22}^b \end{pmatrix} = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}.$$  

We may still see the same two issues in obtaining good reconstructed images. First, the coefficient matrix should be non-singular, i.e.,

$$\begin{vmatrix} -J_{1,c}^1 & J_{2,b}^2 \\ -J_{2,c}^2 & J_{1,b}^1 \end{vmatrix} = |J_1 \times J_2| \neq 0.$$
Therefore, it is required to take two electrical currents, \( J_1 \) and \( J_2 \), in a way that \( J_1 \times J_2 \neq 0 \). Notice that \( J_1 \times J_2 \neq 0 \) is the main requirement to obtain the uniqueness and existence in [20, Definition 2.2] and many other schemes requires the same condition. Reconstructed images are given in Figure 3 using such two current densities. The first two images are obtained using current densities without any noise and are identical to the exact ones. However the next two images, which are reconstructed from 5% noised currents, have vertical and horizontal noise strips. Note that, in an orthotropic resistivity case, the characteristic lines for \( r_{11} \) are vertical lines and for \( r_{22} \) are horizontal ones which is the reason for the noised images.

![Reconstructed orthotropic resistivity images with rectangular network. Exact images are in Figure 7 with color bars.](image)

Since characteristic lines for \( r_{11} \) are vertical and for \( r_{22} \) are horizontal in orthotropic cases, we construct a network system that is aligned in a way to avoid vertical and horizontal directions. Layered and its rotated bricks given in Figure 4 are such resistive network system which were originally introduced in [20]. By using such network systems we may reduce noise strips as obtained in Figure 5. The same 5% noised currents are used in this simulation. Note that the boundary of the circular region is not smooth but has details. One surprising thing is that these details are obtained in recovered images quite well. We will see later that the details are unchanged and clearer for anisotropic cases.

### 3.3. Anisotropic case

An anisotropic resistivity tensor is a symmetric matrix,

\[
\mathbf{r} := \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix}, \quad r_{12} = r_{21}.
\]

In this case the reconstruction relation (2.7) contains three unknowns and hence three equations are required for each cell. Suppose that we have three sets of current densities, \( J_1 \), \( J_2 \) and \( J_3 \). Then, we obtain three equations from each cell,

\[
\begin{align*}
&J_1 \times J_2 \neq 0, \\
&J_1 \times J_3 \neq 0, \\
&J_2 \times J_3 \neq 0,
\end{align*}
\]

where the right sides are given by

\[
g_k := r_{21} J_{1,k}^{1} + r_{22} J_{2,k}^{2} - r_{11} J_{1,k}^{1} - r_{12} J_{2,k}^{2} \quad \text{for} \quad k = 1, 2, 3.
\]

By using matrix multiplication, it is written as

\[
\begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \begin{pmatrix} J_{1,1}^{1} - J_{1,1}^{2} \\ J_{2,1}^{1} - J_{2,1}^{2} \end{pmatrix} = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}.
\]
The same two issues should be resolved to obtain anisotropic resistivity images. However, we will see that overcoming the two issues requires more tricks. First, the coefficient matrix should be non-singular, i.e.,

\[
\begin{vmatrix}
- J_{1,c}^1 & J_{2,b}^1 & J_{1,b}^1 \\
- J_{1,c}^2 & J_{2,b}^2 & J_{1,b}^2 \\
- J_{3,c}^3 & J_{3,b}^3 & J_{3,b}^3
\end{vmatrix} \neq 0.
\]

Therefore, it is required to take three electrical currents, \( J_1, J_2 \) and \( J_3 \), which make this determinant be bounded away from zero if that is possible. However, the second column of the coefficient matrix is more or less the sum of the first and third columns with a negative sign. The only difference is that the values of currents are picked from nearby points. This singularity is not avoided by a choice of the three currents and the singularity becomes even worse as the mesh size becomes smaller. Reconstructed images are given in Figure 6, where the target resistivity images are in Figure 7 with color bars. The pictures in the first row are obtained using the exact current data without any noise. These reconstructed images are almost exact. The images in the second row are obtained using current data with 0.2% noise. The recovered images are severely damaged even with this low noise level. The second issue related to noise propagation along characteristic lines still exist. The characteristic lines for \( r^{11} \) and \( r^{22} \) are still vertical and horizontal. The characteristic lines for \( r^{12} \) are diagonal.

To handle the singularity issue we introduce a triangular network system in the next section which includes diagonal edges as given in Figure 11. The third equation will be produced using this network system. To handle the issue of noise propagation along characteristic lines we will develop a rotation technique. Note that orthotropic resistivity is not orthotropic anymore after a rotation. However, an anisotropic model is still anisotropic after a rotation.
4. Algorithm of anisotropic resistivity reconstruction. In this section we develop a numerical algorithm for anisotropic resistivity reconstruction. This algorithm is based on a local computation and hence the cost is low. We will develop the method and introduce its properties in each step. The basic reconstruction algorithm is from the three equations in (3.3). However, due to the singularity of the coefficient matrix, we introduce an extra network system and replace the third equation. A rotation technique is also introduced to reduce noise propagation along characteristic lines. This technique minimizes blurring effect and gives discontinuity sharply.

4.1. Target resistivity and noises. Target resistivity images are given in Figure 7. For a simpler presentation piecewise constant images are taken. These are the target images of all circular simulation images without a color bar and hence color bars are not repeated in those figures. Let Ω be the whole rectangular domain, Ω₀ be the inside circular subregion, and Ω₁ = Ω \ Ω₀. The values of the target resistivity tensor are

\[
\begin{align*}
    r^{11} &= \begin{cases} 
        16, & x \in \Omega_0, \\
        10, & x \in \Omega_1, 
    \end{cases} \\
    r^{12} &= \begin{cases} 
        12, & x \in \Omega_0, \\
        4, & x \in \Omega_1, 
    \end{cases} \\
    r^{22} &= \begin{cases} 
        20, & x \in \Omega_0, \\
        10, & x \in \Omega_1. 
    \end{cases}
\end{align*}
\]

The eigenvalues of the resistivity tensor in the subregions are

\[
\begin{align*}
    \lambda_1 &= \begin{cases} 
        5.8345, & x \in \Omega_0, \\
        6, & x \in \Omega_1, 
    \end{cases} \\
    \lambda_2 &= \begin{cases} 
        30.17, & x \in \Omega_0, \\
        14, & x \in \Omega_1, 
    \end{cases}
\end{align*}
\]

and corresponding unit eigenvectors are

\[
\begin{align*}
    \mathbf{v}_1 &= \begin{cases} 
        (\mathbf{-0.763, 0.646}), & x \in \Omega_0, \\
        (\mathbf{-0.707, 0.707}), & x \in \Omega_1, 
    \end{cases} \\
    \mathbf{v}_2 &= \begin{cases} 
        (\mathbf{0.646, 0.763}), & x \in \Omega_0, \\
        (\mathbf{0.707, 0.707}), & x \in \Omega_1. 
    \end{cases}
\end{align*}
\]

The principal eigenvector \( \mathbf{v}_1 \) corresponding to the smaller eigenvalue \( \lambda_1 \) is the direction that the electrical current flows more easily. In other words, the muscle fibers are
aligned along the direction of the principal eigenvector $v_1$. Therefore, the piecewise constant target resistivity given above may indicate muscle fibers aligned in directions as given in Figure 8.

![Fig. 7. Target (or exact) resistivity images of $r^{11}$, $r^{12}$ and $r^{22}$ from the left. (The following images keep this order.) They are constant inside the circular subregion $\Omega_0$ and background region $\Omega_1 = \Omega \setminus \Omega_0$.](image)

![Fig. 8. Integral curves of the first eigenvector gives the muscle fiber direction.](image)

In the simulation we use three sets of electrical current densities given in Figure 9. These electrical currents are obtained by solving the forward problem after applying three different boundary currents, respectively, to the anisotropic resistivity body. In the simulation 1% or 5% multiplicative random noise is added to this electrical current data. Precisely, we used

$$n\% \text{ noised current} = \text{exact current} \times \left(1 + \frac{n}{100} X\right),$$

where $X$ is a uniform random variable which has values between one and negative one.

The reconstructed images in Figure 6 are obtained by solving (3.3) using these current densities. We may observe that the algorithm gives correct images if there is no noise in the data. However, the numerical scheme is sensitive to noise which is from the singularity of the coefficient matrix of (3.4). In Figure 10(a) the condition numbers of the coefficient matrix of (3.4) at each grid point are given. The condition numbers are so large and, in particular, the numbers at the two corners are even greater than 1,000, where such values were set as 1,000 in the figure. The first step is to make a system with smaller condition numbers.

4.2. Anisotropic resistivity network with less singularity. We will replace the third equation of (3.3) using a contour integral on a triangular domain. Consider the network system given in Figure 11. Each cell is divided by a diagonal edge. Let $E$ be the triangular subdomain that is bounded by three edges, $a$, $b$ and $c$. Then, the
Fig. 9. Three current set \( J_1, J_2 \) and \( J_3 \) used in the simulations. The first row is the \( x \) component and the second one is \( y \) component.

Fig. 10. Condition numbers of coefficient matrix for anisotropic resistivity reconstruction methods

contour integral on \( E, \oint_{\partial E} rJ(z)dz = 0 \), gives a new relation

\[
-(r_e^{11} \sqrt{2} J^{1,e} + r_e^{12} \sqrt{2} J^{2,e}) - (r_e^{21} \sqrt{2} J^{1,e} + r_e^{22} \sqrt{2} J^{2,e}) + (r_b^{11} J^{1,b} + r_b^{22} J^{2,b}) = f,
\]

where

\[
f = -(r_a^{11} J^{1,a} + r_a^{12} J^{2,a}).
\]

Similarly, we consider the two edges \( b \) and \( e \) are in a same cell by setting

\[
\begin{align*}
r_b^{11} &= r_e^{11}, & r_b^{22} &= r_e^{22}, & r_b^{12} &= r_e^{12} = r_e^{21}.
\end{align*}
\]

Now we apply the relation using the third current density \( J_3 \) and obtained an equation that will replace the third equation of (3.3):

\[
(4.1) \quad -r_b^{11} \sqrt{2} J_3^{1,e} + r_b^{12} (J_3^{1,b} - \sqrt{2} J_3^{2,e} - \sqrt{2} J_3^{1,e}) + r_b^{22} (J_3^{2,b} - \sqrt{2} J_3^{2,e}) = f_3.
\]
In a matrix multiplication form, we may write the system as

\[
\begin{pmatrix}
-J_1^{1,c} & J_1^{1,b} - J_2^{2,c} & J_1^{2,b} \\
-J_2^{1,c} & J_2^{1,b} - J_2^{2,c} & J_2^{2,b} \\
-\sqrt{2} J_3^{1,e} & J_3^{1,b} - \sqrt{2} J_3^{2,e} - \sqrt{2} J_3^{1,e} & J_3^{2,b} - J_3^{2,c}
\end{pmatrix}
\begin{pmatrix}
r_{11} \\
r_{12} \\
r_{22}
\end{pmatrix}
= \begin{pmatrix} g_1 \\
g_2 \\
f_3 \end{pmatrix}.
\]

A picture for condition numbers of this coefficient matrix is given in Figure 10(b). One can clearly see that the condition number is drastically reduced and is now mostly less than 5. The first issue of having nonsingular coefficient matrix is now resolved. Next we use this new system in the following simulations. Reconstructed resistivity tensor images are given in Figure 12 which are much improved in comparison with the ones in Figure 6. Noise levels are increased to 1% or to 5% in these figures.

**4.3. Network Rotation.** The network rotation is designed to construct better images of diagonal components, \(r^{11}\) and \(r^{22}\). The remaining symptoms are the vertical
and horizontal strips in Figure 12, which look similar to the ones of the orthotropic case in Figure 3. The reason for such noise is that the network system is aligned vertically and horizontally, which are the directions of characteristic lines of $r_{11}$ and $r_{22}$, respectively. By rotating our network with an angle $\phi = \pm \pi/12$ we can obtain a network which is not aligned in the characteristic directions of $r_{11}$ and $r_{22}$. Notice that the images of $r_{12}$ in Figure 12 have no such strips and image quality is a lot better than the other two. One may observe wide and light marks in the diagonal direction which appear since noises are mixed and cancelled out each other. This was possible because the network is not aligned in the diagonal direction, which is the direction of characteristic lines of $r_{12}$. The goal is to obtain images of diagonal components which are compatible with off diagonal ones.

We will consider two kinds of rotation technique. The first one is to rotate the network without rotating the resistivity body. If the resistivity body is not of a circular shape, one should make the network system for each rotation angle $\theta$ as in Figure 13(a). Another way is to rotate the network and body together. Then, we may save the effort of making a new network system.

4.3.1. Rotated cells. In this section we consider cells whose edges are not parallel to the $x$ or $y$-axes. In this case the computation is slightly more complicate, but there is no essential difference. Reconstruction relations are still obtained by a contour integral

$$0 = \int_D \nabla \times (rJ) dx = \oint_{\partial D} rJ(z) dz, \quad rJ = \begin{pmatrix} r_{11}J^1 + r_{12}J^2 \\ r_{21}J^1 + r_{22}J^2 \end{pmatrix},$$

where $D$ is the rectangular cell rotated with an angle $\theta$ as given in Figure 13(c). The contour integral gives a relation,

$$-(r_{b}^{11}J^{1,b} + r_{b}^{12}J^{2,b}) \sin \theta + (r_{b}^{21}J^{1,b} + r_{b}^{22}J^{2,b}) \cos \theta$$

$$-(r_{c}^{11}J^{1,c} + r_{c}^{12}J^{2,c}) \cos \theta - (r_{c}^{21}J^{1,c} + r_{c}^{22}J^{2,c}) \sin \theta = g,$$

where

$$g = -(r_{a}^{11}J^{1,a} + r_{a}^{12}J^{2,a}) \cos \theta - (r_{a}^{21}J^{1,a} + r_{a}^{22}J^{2,a}) \sin \theta$$

$$-(r_{d}^{11}J^{1,d} + r_{d}^{12}J^{2,d}) \sin \theta + (r_{d}^{21}J^{1,d} + r_{d}^{22}J^{2,d}) \cos \theta.$$
We consider the resistivity assigned to the two edges \( b \) and \( c \) are unknowns. Set
\[
\begin{align*}
    r_{b}^{11} & = r_{c}^{11}, & r_{b}^{22} & = r_{c}^{22}, & r_{b}^{12} & = r_{c}^{12} = r_{b}^{21} = r_{c}^{21} = r_{b}^{c},
\end{align*}
\]
i.e., the two edges are placed in a same cellular plate. Now we obtain an equation corresponding to (2.7) in terms of the edge index \( b \),
\[
(4.3) \quad -r_{b}^{11}(J_{1}^{1,b} \sin \theta + J_{1}^{1,c} \cos \theta) + r_{b}^{12}((J_{1}^{1,b} - J_{2}^{2,c}) \cos \theta)
- (J_{2}^{2,b} + J_{1}^{1,c}) \sin \theta) + r_{b}^{22}(J_{2}^{2,b} \cos \theta - J_{2}^{2,c} \sin \theta) = g.
\]

Let \( J_{1}, J_{2} \) and \( J_{3} \) be three given currents. We will use the above relation to \( J_{1} \) and \( J_{2} \). Remember that we should not apply the same relation to all three current data due to the singularity of the coefficient matrix. Let \( E \) be the triangular subdomain bounded by three edges, \( a, b \) and \( c \). Then, the contour integral on \( \partial E \) gives another relation,
\[
\sqrt{2}(r_{c}^{11} J_{1}^{1,c} + r_{c}^{12} J_{2}^{2,c})(- \cos \theta + \sin \theta) + \sqrt{2}(r_{c}^{21} J_{1}^{1,c} + r_{c}^{22} J_{2}^{2,c})(- \cos \theta - \sin \theta)
- (r_{b}^{11} J_{1}^{1,b} + r_{b}^{12} J_{2}^{2,b}) \sin \theta + (r_{b}^{21} J_{1}^{1,b} + r_{b}^{22} J_{2}^{2,b}) \cos \theta = f,
\]
where
\[
f = -(r_{a}^{11} J_{1}^{1,a} + r_{a}^{12} J_{2}^{2,a}) \cos \theta - (r_{a}^{21} J_{1}^{1,a} + r_{a}^{22} J_{2}^{2,a}) \sin \theta.
\]

Similarly, we consider the resistivity assigned to the two edges \( b \) and \( c \) are unknowns. Set
\[
\begin{align*}
    r_{b}^{11} & = r_{c}^{11}, & r_{b}^{22} & = r_{c}^{22}, & r_{b}^{12} & = r_{c}^{12} = r_{b}^{21} = r_{c}^{21} = r_{b}^{c},
\end{align*}
\]
i.e., the two edges are placed in a same cellular plate. Now we obtain an equation corresponding to (4.1) in terms of the edge index \( b \),
\[
(4.4) \quad -r_{b}^{11}(J_{1}^{1,b} \sin \theta + \sqrt{2}J_{1}^{1,c}(\cos \theta - \sin \theta)) + r_{b}^{22}(J_{2}^{2,b} \cos \theta - \sqrt{2}J_{2}^{2,c}(\sin \theta + \cos \theta))
+ r_{b}^{12}(J_{1}^{1,b} - \sqrt{2}J_{2}^{2,c}) \cos \theta - (J_{2}^{2,b} + \sqrt{2}J_{1}^{1,c} - \sqrt{2}J_{2}^{2,c}) \sin \theta) = f.
\]
This relation is applied to the third current data \( J_{3} \) and completes the reconstruction algorithm.

The network system in Figure 13(a) is a case with a rotating angle \( \theta = -\frac{\pi}{4} \). In this case the cells are aligned diagonally. Therefore, we may expect the images of \( r_{11} \) and \( r_{22} \) would be better than the one for \( r_{12} \). The reconstructed images in Figure 14 are obtained using the rotated cells. However, we did not get clear improvement in these cases.

4.3.2. Rotated resistivity body. Rotating the cells of a resistivity network requires a new discretization and costs extra efforts. Now we consider a method which does not require a new discretization, but costs extra linear algebra. The basic idea is to rotate the resistivity body and the network together as shown in Figure 13(b). In this way we may use the same network system and the same current data. However, we may obtain a meaningful improvement in the image quality.

Since the body is rotated with the network, the current vector field should be also rotated. Denote the rotation tensor by
\[
(4.5) \quad R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix},
\]
where the rotation angle is denoted by $\theta$. Then, the rotated current field by the angle $\theta$ is

$$\tilde{J} = R(\theta)J.$$  

The resistivity tensor $\mathbf{r}$ under the new coordinate system is

$$\tilde{\mathbf{r}} = R(-\theta)^t \mathbf{r} R(-\theta),$$

or

$$\begin{align*}
\tilde{r}^{11} &= \cos^2 \theta r^{11} - 2 \sin \theta \cos \theta r^{12} + \sin^2 \theta r^{22}, \\
\tilde{r}^{12} &= \cos \theta \sin \theta r^{11} + (\cos^2 \theta - \sin^2 \theta)r^{12} - \cos \theta \sin \theta r^{22}, \\
\tilde{r}^{22} &= \sin^2 \theta r^{11} + 2 \sin \theta \cos \theta r^{12} + \cos^2 \theta r^{22}.
\end{align*}$$

(4.6)

Therefore, the curl free equations and boundary conditions are written as

$$\begin{align*}
\nabla \times (\tilde{\mathbf{r}} \tilde{\mathbf{J}}_k) &= 0 \text{ in } \tilde{\Omega}, \ k = 1, 2, 3, \\
\tilde{\mathbf{r}} &= \tilde{\mathbf{r}}_0 \text{ on } \tilde{\Gamma} \subset \partial \tilde{\Omega},
\end{align*}$$

(4.7)
where the boundary resistivity is also given by

\[ \tilde{r}_0 = R(-\theta)^t r_0 R(-\theta). \]

The characteristic lines for \( \tilde{r}_{11} \) and \( \tilde{r}_{22} \) are vertical and horizontal lines under the new coordinate system and the ones for \( \tilde{r}_{12} \) are diagonal ones. However, the network is aligned according to the old coordinate system and is tilted by an angle \( \theta \). Therefore, we have a chance to get the noise of \( \tilde{r}_{11} \) or of \( \tilde{r}_{22} \) cancelled out. One may obtain various quality of images depending on the choice of rotation angle \( \theta \). The recovered resistivity images in Figure 15 are obtained using angles \( \theta = -\pi/12 \) and \( -\pi/12 \). The image of \( \tilde{r}_{11} \) is better with \( \theta = -\pi/12 \) and for \( \tilde{r}_{22} \) is with \( \theta = \pi/12 \).

Now we return to resistivity images in the original coordinates using some of better images of \( \tilde{r}^{ij} \). The image of \( r_{12} \) is taken from Figure 12(b). The images of \( r_{11} \) and \( r_{22} \) are constructed by solving (4.8). These are the final images of the algorithm in this paper. A quality comparison of reconstructed images is given in Figure 19.

**Remark 1 (Choice of angles).** If the rotation angle is \( \theta = \pi/4 \), then the role of \( r_{11} \) and \( r_{22} \) are simply swapped. Hence, the meaningful range of rotation angle is of size \( \pi/4 \). In the algorithm we have chosen three angles \( \theta = -\pi/12, 0, \pi/12 \), which divide an interval of length \( \pi/4 \) with equal distances.

### 4.4. Denoising and Enhancing

In this section denoising and enhancing techniques are used, which are applied to noised current data and to reconstructed images, respectively. First we regularize noised current densities. A regularization process usually helps us to obtain less oscillatory images. However, some of information is lost in the process. Regularization is a local smoothing process that reduces the variation
Fig. 17. Reconstructed images obtained using denoised current densities.

Fig. 18. Enhanced images of $r^{22}$ by a total variation minimizing technique (4.10)-(4.11). The first row are enhanced images of Figure 12 and the second row are of Figure 16. However, the interfaces are blurred. The parameter $\sigma^2$ is the variation.

of data by averaging the data using a weight kernel. We use a typical kernel matrix,

\[
\begin{pmatrix}
0.0251 & 0.1453 & 0.0251 \\
0.1453 & 0.3183 & 0.1453 \\
0.0251 & 0.1453 & 0.0251 \\
\end{pmatrix}
\]

The images in the Figure 17 are reconstructed ones using such regularized current data. The noise strips are regularized in comparison with the images in Figure 12. However, the circular boundary of the discontinuity is also blurred.

Next we apply the total variation minimizing technique to enhance the reconstructed images. We use the method introduced in [33]. In the algorithm a degenerate parabolic problem,

\[
(4.10) \quad u_t = \nabla \cdot \left( \frac{\nabla u}{|\nabla u|} \right) - \zeta (u - u_0), \quad t > 0, x \in \Omega,
\]

is solved until the solution reaches a steady state. The initial value $u(x, 0) = u_0(x)$ is a two dimensional image obtained previously and the boundary condition is the usual
zero flux condition across the boundary. The parameter $\zeta$ is given by

\begin{equation}
\zeta = -\frac{1}{2\sigma^2} \int_{\Omega} \left[ |\nabla u| - \left( \frac{\nabla u_0 \cdot \nabla u}{|\nabla u|} \right) \right] dx.
\end{equation}

First, the image of $r_{22}$ in Figure 12 is enhanced using three variation values of $\sigma^2$ and the enhanced images are given in the first row of Figure 18. We may observe that the noise strips are weakened as variation $\sigma^2$ increases, but the circular boundary is also blurred in the process. Remember that the images obtained by the rotation technique, Figure 16, make the noise strips weakened, but keeps the interface sharp. Next we take this better image in Figure 16 as an initial value and enhance it similarly, which are given in the second row of Figure 18. As the variation $\sigma^2$ increases the discontinuity circular edge becomes smeared.

4.5. Comparison of images and algorithms. For a comparison of image qualities and algorithm properties the reconstructed images of the component $r_{22}$ are collected in Figure 19. These reconstructed images are obtained using current densities with 5% multiplicative noise. An except is Figure 6(f) which is obtained with 0.2% noised data. The method used for Figure 6(f) is unstable to noise and replacing the third row of the equation in (3.4) and obtain (4.2) gives a key improvement. This process resolved the singularity issue of the algorithm and allowed us to obtain a meaningful images in Figure 12. However, noise strips along characteristic lines remain and the rest of the algorithm is to reduce them.

The rotation technique in Section 4.3, which rotates the network and the resistivity body together, reduces the noise strips considerably as given in Figure 16. A remarkable property of this process is that the circular discontinuity edge remains sharply including details and only the noise strips are weakened. These are final images that the method of this paper provides. Figure 14 is obtained by rotating the cells without rotating the body. However, the image quality is not improved. De- noising or enhancing techniques, Figure 17 and Figure 18, reduce the noise. However, these methods blur the noise and interface together.

5. Numerical tests. In this section we further test the introduced anisotropic resistivity tensor imaging algorithm for two cases. One is designed to demonstrate that the reconstructed anisotropic resistivity images provide the muscle fiber structure of the human body which contains continuous and discontinuous variations together. The other one is designed to introduce the challenges in anisotropic resistivity tensor imaging which is the discontinuity of muscle fiber directions, but not simply the discontinuity of resistivity images. We will see that, if the direction of the first eigenvector turns discontinuously with an angle $\phi = \frac{\pi}{2}$ (or perpendicularly) across an interface, current density becomes singular and the performance of a reconstruction method drops.

5.1. Muscle fiber imaging. The main reason why an animal body or brain has anisotropic resistivity is in its muscle and nerve fiber structure. If a resistivity body has a fiber structure, the electricity flows more easily in the fiber direction. The muscle fiber direction may change continuously or discontinuously (see a cartoon in Figure 21(a)). In this section we consider the target anisotropic resistivity tensor given in the first row of Figure 20. Let $0 < \lambda_1 \leq \lambda_2$ be two eigenvalues and $v_1$ and $v_2$ be corresponding eigenvectors. Anisotropic resistivity tensors are symmetric and positive. Therefore, the eigenvalues are real and positive valued. Note that the first eigenvector $v_1$ is pointing the direction that the electrical current flows more easily,
Image quality comparison. Currents with 5% multiplicative noise are used except the ones for Figure 6(f) which are with only 0.2% noise.

Discontinuously and continuously varying anisotropic resistivity. The exact target images are given in the first row. The ones in the second row are obtained using the rotation method with angles $\theta = \frac{\pi}{12}$ and $-\frac{\pi}{12}$. Current densities with 5% multiplicative noise are used.

i.e., the direction of muscle fibers. The integral curves of the first eigenvector are displayed in Figure 21(b).

We reconstruct the anisotropic images using the reconstruction method constructed in the previous section. We first solved a forward problem to produce three sets of electrical currents and added a 5% multiplicative noise. We have solved the system consists of (4.3) for $k = 1, 2$ and (4.4) for $k = 3$. We solved the system for three rotation angles $\theta = \frac{\pi}{12}, 0$ and $-\frac{\pi}{12}$. The image of $r^{12}$ is taken from the ones with $\theta = 0$ and the images of $r^{11}$ and $r^{22}$ are obtained using the relations in (4.8). The reconstructed images are given in the second row of Figure 20. These simulation results
are the ones corresponding to the ones in Figure 16. The integral curves for the first eigenvector are displayed in Figure 21(c). These figures show how the reconstruction method recovers the muscle fiber direction.

5.2. Discontinuity in the first eigenvector. So far we have observed how the discontinuity in resistivity image is reconstructed. For example, the images in Figure 16 show the circular interface of discontinuity sharply. However, the real challenge in anisotropic resistivity reconstruction arises when the principal eigenvector, \( \mathbf{v}_1 \), but not simply resistive images, has discontinuity with an angle close to \( \frac{\pi}{2} \).

In this section we consider three target anisotropic resistivity tensor distributions, where \( \Omega \) is the whole rectangular domain, \( \Omega_0 \) is the inside circular subregion, and \( \Omega_1 = \Omega \setminus \Omega_0 \). The resistivity values in the exterior domain \( \Omega_1 \) are fixed for the three cases, which are

\[
(5.1) \quad r_{11} = 5, \quad r_{12} = 2, \quad r_{22} = 5, \quad x \in \Omega_1.
\]

The eigenvalues and eigenvectors in the exterior domain \( \Omega_1 \) are

\[
\lambda_1 = 3, \quad \lambda_2 = 7, \quad \mathbf{v}_1 = (-0.7071, 0.7071), \quad \mathbf{v}_2 = (0.7071, 0.7071), \quad x \in \Omega_1.
\]

The resistivity tensor in the interior circular domain is obtained by rotating the resistivity tensor of the exterior domain by angles \( \phi = \frac{\pi}{6}, \frac{\pi}{3}, \) and \( \frac{\pi}{2} \), i.e., for \( x \in \Omega_0 \), the
components are given by
\[(5.2)\]

\[r^{11} = \begin{cases} 
6.7321, & \text{for } \phi = \pi/6, \\
6.7321, & \text{for } \phi = \pi/3, \\
5, & \text{for } \phi = \pi/2, 
\end{cases} \]

\[r^{12} = \begin{cases} 
1, & \text{for } \phi = \pi/6, \\
-1, & \text{for } \phi = \pi/3, \\
-2, & \text{for } \phi = \pi/2, 
\end{cases} \]

\[r^{22} = \begin{cases} 
3.2679, & \text{for } \phi = \pi/6, \\
3.2679, & \text{for } \phi = \pi/3, \\
5, & \text{for } \phi = \pi/2, 
\end{cases} \]

\[x \in \Omega_0.\]

The eigenvalues in the interior domain are the same as the exterior ones since the interior anisotropic tensor is obtained by simply rotating the exterior one. The eigenvectors are in the interior domain are

\[v_1 = \begin{cases} 
(-0.2588, 0.9659), & \phi = \pi/6, \\
(0.2588, 0.9659), & \phi = \pi/3, \\
(0.7071, 0.7071), & \phi = \pi/2, 
\end{cases} \]

\[v_2 = \begin{cases} 
(0.9659, 0.2588), & \phi = \pi/6, \\
(-0.9659, 0.2588), & \phi = \pi/3, \\
(-0.7071, 0.7071), & \phi = \pi/2, \end{cases} \]

\[x \in \Omega_0.\]

For each three cases, \(\phi = \frac{\pi}{6}, \frac{\pi}{3}\) and \(\frac{\pi}{2}\), three currents in each column are used for each case. Singularity increases as the contact angle \(\phi\) approaches \(\pi/2\).

![Fig. 22. The magnitude of current densities \(|J_{k,\phi}|\) for \(k = 1, 2, 3\) and \(\phi = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\). Three currents in each column are used for each case. Singularity increases as the contact angle \(\phi\) approaches \(\pi/2\).](image)

The blowup phenomenon for isotropic case usually happens when two highly conductive material approaches. However, for an anisotropic case, a different kind of singularity appears which gives an extra challenge in anisotropic resistivity reconstruction.
(a) The direction of muscle fiber of interior and exterior domain has angle \( \phi = \frac{\pi}{6} \).

(b) The direction of muscle fiber of interior and exterior domain has angle \( \phi = \frac{\pi}{3} \).

(c) The direction of muscle fiber of interior and exterior domain has angle \( \phi = \frac{\pi}{2} \).

Fig. 23. Images of anisotropic tensor components \( r_{11} \), \( r_{12} \) and \( r_{22} \). They are obtained after rotating the resistivity body and the network with angles \( \theta = \frac{\pi}{12} \) and \( -\frac{\pi}{12} \), respectively. Current densities with 5% multiplicative noise are used.

The resistivity images given in Figure 23 are obtained rotating the network and body together as in Section 4.3. The image quality becomes worse as the contact angle \( \phi \) increases to \( \frac{\pi}{2} \). For example we may see the circular domain relatively clearly for the case with \( \phi = \frac{\pi}{6} \) from all of the three images. For the case with \( \phi = \frac{\pi}{3} \) the lower part of the circular domain for \( r_{22} \) is not clear. For the case with \( \phi = \frac{\pi}{2} \) the image of \( r_{12} \) suffers from the noise along the diagonal direction. The other two images of \( r_{11} \) and \( r_{22} \) should be constant ones, but there are circular domains which do not exist in the exact resistivity images.

REFERENCES


1. **Electrical Impedance Tomography**

2. **Anisotropic Conductivity**

3. **Reconstruction of Small Inhomogeneities**

4. **Source Reconstruction**

5. **Measurement of Electrical Current Density**

6. **Uniqueness and Convergence**

7. **Virtual Resistive Network**

8. **Orthotropic Conductivity**

9. **Reconstruction of Conductivity**

10. **Inverse Anisotropic Diffusion**
    - Inverse anisotropic diffusion from power density measurements in two dimensions, Inverse Problems, 28 (2012), pp. 084001, 20,


