Homework 1

MAS501 Analysis for Engineers, Spring 2011

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1. A real number r is said to be *algebraic* if r is a root of a polynomial with rational coefficients. If r is not algebraic, r is said to be *transcendental*. Show that the set of all transcendental numbers is uncountable. (It is known that e and π are transcendental.)

   Hint: Use the fact that the set of all polynomials with rational coefficients is countable. You don't need to write down the proof of this fact; just try to understand why it is true.

2. Let (Ω_1, d_1) and (Ω_2, d_2) be metric spaces.

   (a) Prove that $(\Omega_1 \times \Omega_2, d)$ is a metric space, where d is defined by the formula

   $$d[(x_1, x_2), (y_1, y_2)] = d_1(x_1, y_1) + d_2(x_2, y_2).$$

   The space $(\Omega_1 \times \Omega_2, d)$ is called the *product metric space*.

   (b) Let $\mathbf{R} \times \mathbf{R}$ be the product metric space constructed from the Euclidean space $\mathbf{R}$. Then obviously the metric of $\mathbf{R} \times \mathbf{R}$ is different from the metric of Euclidean space $\mathbf{R}^2$. Although, we can prove that any open set of one metric space is also an open set of the other metric space. (This means their topology are same; they have same "nearness".) Prove it.