Homework 2

MAS501 Analysis for Engineers, Spring 2011

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1. (Discrete metric) Let Ω be a set.

   (a) Prove that a function $d : \Omega \times \Omega \to \mathbf{R}$ defined by

   $$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$$

   is a metric on Ω. The metric d is called the *discrete metric* for Ω.

   (b) Let (Ω, d) be a metric space with discrete metric. Find all the isolated points of Ω. Now can you see why the metric is called *discrete*?

2. (Set operations and compactness)

   (a) Let $K_1, \cdots, K_n$ be a finite collection of compact subsets of a metric space Ω. Prove that $\cup_{i=1}^{n} K_i$ is compact. Show (by example) that this result does not generalize to infinite unions.

   (b) Let I be an index set and $\{K_i : i \in I\}$ be a collection of compact subsets of a metric space Ω. Prove that $\cap_{i \in I} K_i$ is compact.

   (c) Let K be a compact subset of Euclidean space $\mathbf{R}$ and let $y \in \mathbf{R}$. Prove that the set $\{x + y : x \in K\}$ is compact.