# Homework 3

MAS501 Analysis for Engineers, Spring 2011

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1. Let $A$ and $B$ be nonempty sets of real numbers and assume $\sup A$ and $\sup B$ are finite. Prove that

   $$\sup(A+B) = \sup A + \sup B,$$

   where

   $$A + B = \{a + b : a \in A, b \in B\}.$$

   What can we say when either of $\sup A$ or $\sup B$ is *not* finite?

2. (Product metric space and Cauchy sequence) Let $\Omega_1$ and $\Omega_2$ be metric spaces and $\Omega_1 \times \Omega_2$ be the product metric space. (For the definition of product metric space, refer to Homework #1.)

   (a) Prove that if a sequence $\{(x_n, y_n)\}$ is Cauchy in $\Omega_1 \times \Omega_2$, then the sequence $\{x_n\}$ is Cauchy in $\Omega_1$ and the sequence $\{y_n\}$ is Cauchy in $\Omega_2$. Is the converse true?

   (b) Prove that if $\Omega_1$ and $\Omega_2$ are complete, then the product metric space $\Omega_1 \times \Omega_2$ is also complete. Is the converse true?