

Homework 7

MAS501 Analysis for Engineers, Spring 2011

1. Suppose that f'' exists on $[0, 1]$ and that $f(0) = f(1) = 0$. Suppose also that

$$|f''(x)| \leq K \quad \text{for } x \in (0, 1).$$

Prove that

$$\left| f' \left(\frac{1}{2} \right) \right| \leq \frac{K}{4}$$

and

$$|f'(x)| \leq \frac{K}{2} \quad \text{for } x \in [0, 1].$$

2. Prove that if f is a continuous nonnegative function on $[a, b]$ and

$$\int_a^b f(x) dx = 0,$$

then $f(x) = 0$ for all x in $[a, b]$.