

Homework 8

MAS501 Analysis for Engineers, Spring 2011

1. Let f be continuous on $[a, b]$ and α be a jump function having jumps at the points x_1, x_2, \dots, x_N .

(a) Prove that $\int_a^b f d\alpha$ exists.

Hint: Use Theorem 6.1.2 in the textbook.

(b) Show that

$$\int_a^b f d\alpha = \sum_{n=1}^N f(x_n) c_n,$$

where $c_n := \alpha(x_n^+) - \alpha(x_n^-)$, $1 \leq n \leq N$.

2. Give an example of a function α which is continuous on $[0, 1]$ and differentiable on $(0, 1)$ such that $\alpha \in BV[0, 1]$, but α' is unbounded on $(0, 1)$.