Homework 10

MAS501 Analysis for Engineers, Spring 2011

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1. Let f and g be bounded measurable functions defined on a set E of finite measure. Prove the following statements:

   (a) If $f \leq g$ a.e., then
   $$\int_E f \leq \int_E g.$$

   (b) It holds that
   $$\Big|\int_E f\Big| \leq \int |f|.$$

   (c) If $A \leq f(x) \leq B$ a.e., then
   $$A\, m(E) \leq \int_E f \leq B\, m(E).$$

2. Find examples showing the following statements are true:

   (a) We may have strict inequality in Fatou's Lemma.

   (b) Monotone Convergence Theorem need not hold for decreasing sequence of functions.

3. Let f be a nonnegative measurable function. Show that $\int f = 0$ implies $f = 0$ a.e.