New function space in general unbounded domains and its applications to the Navier-Stokes equations

Hideo Kozono
Waseda University (kozono@waseda.jp)

We consider problems on the mathematical fluid mechanics in general unbounded domains Ω with non-compact boundaries. In such domains Ω, it is known that the usual Helmholtz decompositions in $L^r(\Omega)$, $1 < r < \infty$ does not hold except $r = 2$, and hence we need to introduce another function space $\tilde{L}^r(\Omega)$ defined by $\tilde{L}^r(\Omega) = L^r(\Omega) + L^2(\Omega)$ for $1 < r \leq 2$ and $\tilde{L}^r(\Omega) = L^r(\Omega) \cap L^2(\Omega)$ for $2 \leq r < \infty$, respectively. This new function space $\tilde{L}^r(\Omega)$ plays a substitutionary role for $L^r(\Omega)$ so that the Helmholtz decomposition holds. Defining the Stokes operator A in $\tilde{L}^r(\Omega)$, we can develop well-known techniques like analyticity of the semi-group $\{e^{-tA}\}_{t>0}$ and the maximal regularity theorem on $\partial_t + A$ as well as the characterization of the domains of the fractional powers A^α, $0 < \alpha < 1$. As applications of the theory on $\tilde{L}^r(\Omega)$, we are able to treat fundamental problems on uniqueness, regularity and decay properties of weak solutions of the Navier-Stokes equations in Ω.

Contents