MATHEMATICAL BIOLOGY

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1. The PDE's of Cell Biology - Making Diffusion Your Friend

Diffusion is the enemy of life. This is because diffusion causes small particles to spread out, and for aggregates of particles to dissipate. Thus, in order to be alive and maintain its structure, an organism must have ways to counteract the constant tendency of things to spread out. And indeed they do. Plants, for example, are able to harness the energy of the sun to convert carbon dioxide and water into high energy compounds such as carbohydrates. These high energy compounds are then carefully deconstructed by living organisms to do work moving things around and building and repairing their structures. In this way, living things are able to combat the tendency of structures to dissipate and fall apart.

However, living organisms do much more than simply counteract diffusion; they actually exploit it for specific purposes. That is, they expend energy to concentrate molecules and then use the fact that molecules move by diffusion down their concentration gradient to do useful things. How do they actually do this? The short answer is that they couple diffusion with appropriate chemical reactions and are thereby able to exploit the inherent diffusive motion of molecules. Indeed, many of the processes that take place in living cells can be described as the interaction of reacting and diffusing chemical species. This realization has led to the description of many interesting biological processes in the language of partial differential equations and in turn what has been learned about these partial differential equations has led to an increased understanding of how biological systems work.

In this talk, I give several examples of the ways that cells use diffusion to their advantage, and describe the partial differential equations that model these processes. In particular, I will describe how molecular diffusion and reaction are used to make signals, to create functional aggregates, to take a census, and to make length measurements.

In this way, I hope to convince you that living organisms have made diffusion their friend, not their enemy, and in the process, demonstrate the importance of understanding the solutions of the partial differential equations governing diffusion-reaction processes.

2. Introduction to Cardiac Electrophysiology - Excitability and heart attacks

What is similar about football fans, a forest fire and a heart attack? The answer: Certain features of their behavior have a common mathematical description. As a result we can learn something about heart attacks from the behavior of crazed football fans.

A heart attack occurs when there is an occlusion of a coronary artery leading to tissue damage. A heart attack is fatal when there is a subsequent disruption of the normal electrical signal of the heart, leading to fibrillation.

The purpose of this talk is derive the equations governing the dynamics of electrical activity of the heart and to show how an understanding of the solutions of these equations can help us understand aspects of cardiac arrhythmias, what they are, how they occur, and how they might be eliminated or prevented.

3. The Dynamics of Multiphase BioFluids

Biogels are complex polymeric networks whose proper function is important to many physiological processes. For example, the proper function of mucus gel is important for airway clearance, reproduction, digestion, gastric protection, and disease protection and its failure is involved in cystic fibrosis, gastric ulcers, and reproductive dysfunction.

Many complex biofluids can be thought of as consisting of two or more comingled phases with distinct physical/rheological properties. In this talk, I will give an introduction to the mathematics of multiphase biofluids, showing the equations of motion and some of the important features of their solutions. Included will be a description of emergent properties of these equations such as pattern formation, phase separation, osmotic swelling, biofilm growth, vesicle formation and phase separation on 2 dimensional membranes, and vesicle exocytosis.

4. The Dynamics of Gelation and Fibrin clot formation

Fibrin clots are gel structures in the blood that are crucial for prevention of bleeding after injury, but inappropriate formation of clots can be life threatening, as it is implicated in hearts attacks and strokes. There are three phases of clot dynamics that are important to their biological function. These are their formation (e.g., blood clotting), degradation (clot dissolution), and swelling/deswelling kinetics (during mucin secretion/exocytosis, for example).

The purpose of this talk is to describe recent advances in the study of the dynamics of fibrin clot formation. In particular, I will derive and discuss features of a new partial differential equation model that describes the growth of fibrin clots as a polymerization/gelation reaction. The solution of this PDE model gives insight into the branching structure of clots that are formed under various physiological conditions. As an introduction, I will describe a recent extension of the Ziff model of polymer gel formation that models the spatial-temporal process of polymer gel formation.

5. The Huxley equation - How to pull with a burning rope and other stories about molecular motors

The movement and force generation of muscle fibers, the translocation of viruses and macromolecules by molecular motors and the segregation of chromosomes during cell division are three examples of active processes that turn chemical energy into mechanical movement. The purpose of this talk is to show how Huxley-type models (i.e. generalizations of Huxley's model of muscle crossbridge kinetics, a reaction-advection equation) are used to help us understand features of molecular motors such as dyneins and kinesins as well as less familiar depolymerization motors that are used to pull the chromosomes to opposite poles of a dividing cell.