10th Korea PDE Winter School

February 17th–21st, 2020, NIMS, Daejeon, Korea

1) Principal Speakers

Regularity theory for uniformly degenerate elliptic equations

Qing Han(Notre DAME, USA)

Lecture 1: Review of Schauder theory; introduction of uniformly degenerate elliptic equations.

Lecture 2: Solvability of the Dirichlet problem and lower regularity.

Lecture 3: Obstruction to higher regularity and modifications.

Lecture 4: The Leowner-Nirenberg problem.

Lecture 5: The minimal surface equation in the hyperbolic space.

An introduction to hyperbolic system of conservation laws

Bongsuk Kwon (UNIST) and Moon-jin Kang (Sookmyung women's University)

Lecture 1: Formation of singularities for 1D nonlinear hyperbolic PDEs.(By B. Kwon)

ABSTRACT

In this lecture, we first discuss some basic properties of nonlinear hyperbolic systems. These include the finite speed of propagation, symmetrizability in terms of the existence of entropy-flux pair. Especially we study the Riemann invariants for the 1D hyperbolic system. Then we will go over Peter Lax's singularity formation theory for 1D nonlinear hyperbolic systems using the Riemann invariants. After introducing Lax's general theory, we carefully look into the isentropic compressible Euler equations as an example and discuss the formation of singularity and global existence.

Lecture 2: Well-posedness theory for entropy solutions to compressible Euler system(By M. Kang)

ABSTRACT

The well-posedness theory for entropy solutions to compressible Euler system remains open wide even for 1-D case. For 1-D theory, we introduce the Lax theory (1957) on the Riemann problem, that is, on the existence and uniqueness of entropy solutions starting from a simple initial data, which is a jump discontinuity with small amplitude, in the class of self-similar solutions. The unique self-similar solution usually consists of three different types of simple waves: shock wave; rarefaction wave; contact discontinuity. Those waves will be explicitly handled in this lecture. However, without the self-similarity or the smallness of initial data, the non-uniqueness (thus, ill-posedness) of entropy solutions for the Riemann problem was proved by the convex integration method (due to De Lellis and Szekelyhidi).

Lecture 3: Formation of singularities for multi-D compressible Euler equations.(By B. Kwon)

ABSTRACT

In this lecture, we will first discuss the C^1 blow-up for the multi-D compressible Euler equations following the work of T. Sideris. For simplicity, we shall mostly focus on the large data blow-up result: when the smooth initial data satisfies the "supersonic" condition, we will see that the C^1 solutions cannot exist globally by introducing an averaged quantities. Despite this beautiful result, not much has been known about what really happens upon the blow-up time for a long time. Recently the formation of interesting singularities (more specifically, shocks with non-zero curl) was proved in a constructive way by T. Buckmaster, S. Shkoller and V. Vicol, for the 2D and 3D compressible Euler equations. We will briefly discuss their results and key ideas of the proofs. If time permits, we will talk about some open problems concerning the formation of singularities vs. the global existence for the Euler-Poisson systems, which arise in plasma physics.

Lecture 4: Uniqueness of entropy solutions to compressible Euler system in the class of vanishing viscosity limits.(By M. Kang)

ABSTRACT

In this lecture, we discuss about the conjecture: uniqueness of entropy solutions to compressible Euler system in the class of vanishing viscosity limits from the associate Navier-Stokes system (as a physical viscous system for the Euler). The conjecture was proved by Bianchini and Bressan (2005), in the class of vanishing "artificial" viscosity limits. We provides a recent result for the conjecture in the class of vanishing "physical" viscosity limits from the Navier-Stokes system. This is based on a collaboration with Alexis Vasseur.

2) Invited Speakers

Stability of traveling waves in a hyperbolic-parabolic system arising from a chemotaxis model

Kyudong Choi (UNIST)

ABSTRACT

We consider a simplified chemotaxis model of tumor angiogenesis, described by the Keller-Segel system with logarithmic singular sensitivity with linear consumption rate. The system is known to allow traveling wave solutions (so-called viscous shocks) whose stability has recently received both mathematical and biological interest. In this talk, we study this stability question in two different settings. First, by using a relative entropy functional of the system, which can capture how close a solution at a given time is to a given shock wave in almost L2-sense, the functional is NON-increasing for all time when the shock strength is small enough and it is away from zero-level. Second, stability of PLANAR traveling waves is obtained in two dimensional infinite cylindrical domain by using so-called anti-derivative method with weighted energy estimates allowing zero boundary condition on one end for shocks. This talk is based on joint work with M. Kang, Y. Kwon, A. Vasseur and M. Chae, K. Kang, J. Lee.

Asymptotic analysis for a Vlasov-Fokker-Planck/Navier-Stokes system

Young-Pil Choi (Yonsei University)

ABSTRACT

Recently, the study on particle-fluid system is gathering a lot of attentions due to their applications, for example, in the study of sedimentation phenomena, fuel injector in engines, and compressibility of droplets of the spray, etc. In this talk, we consider a coupled kinetic-fluid system consisting of the nonlinear Vlasov-Fokker-Planck equation and compressible Navier-Stokes equations in a bounded domain. For that kinetic-fluid system, we discuss the hydrodynamic limit corresponding to the strong noise and local velocity-alignment force.

TBA

Hyungju Hwang (Postech)

ABSTRACT

TBA

Stability of periodic traveling waves

Soyeon Jung (Kongju National University)

ABSTRACT

In this talk, we consider stability of periodic traveling waves for systems of reaction diffusion equations. In particular, we study the rigorous proof of spectral stability of periodic solution bifurcating from the uniform state of Brusselator model. Moreover, we briefly talk about the amplitude equation of periodic solutions.

Accelerated Douglas-Rachford Operator Splitting Method for Variational Problems

Donghwan Kim (KAIST)

ABSTRACT

Operator splitting methods, such as Douglas-Rachford method, are often preferred for efficiency, when solving variational problems that are equivalent to finding a zero of the sum of two monotone operators. Douglas-Rachford method was originally developed as an alternating direction implicit (ADI) method for numerically solving the heat equation, and later was extended by Lion and Mercier for finding a zero of the sum of two monotone operators. For computational efficiency, this talk presents an accelerated Douglas-Rachford method that has a fast rate of convergence in terms of the fixed-point residual. Numerical experiments on variational problems will be presented.

Statistical Models for Longitudinal Study in Medical Research

Soon-Sun Kwon (Ajou University)

ABSTRACT

In medical research, there exist a variety of data structures such as serial data sets with different follow-up intervals and points, longitudinal data sets with missing values, data sets with multiple measurements for individual subjects (for example, measurements from left and right sides), and so on. Specially, longitudinal data are used in statistical studies that accept many repeated measurements as well as the different time spans of the measurements between or within subjects. Furthermore, correct inferences can particularly be obtained by considering the correlation between repeated measurements within subjects. In this talk, I introduce statistical prediction models for handling longitudinal data in medical research.

Introduction to deep learning in medical image restoration

Hyung Suk Park (NIMS)

ABSTRACT

In X-ray computed tomography (CT), severe artifacts (e.g., noise, streaking and shadowing artifacts) occur due to low-dose or high-attenuation objects, scattering, and so on. These artifacts in CT images decrease the reliability of diagnosis. Recently, deep learning techniques have shown great potential for artifacts reduction in the field of image processing. Most of these approaches learn the relationship between artifact images and artifact-free (i.e., ground-truth) images. However, such paired training data are not generally available in clinical practice. In this talk, I will introduce simulation-based and unpaired learning methods, which can be used to circumvent such issue.

TBA

Seungjin Ryu (University of Seoul)

ABSTRACT

TBA

Quantum classical correspondence between Energy-Casimir minimizers of the gravitational Hartree and Vlasov-Poisson equations.

Jinmyoung Seok (Kyonggi University)

ABSTRACT

For the gravitational Vlasov-Poisson equation, Guo and Rein constructed a class of classical isotropic states as minimizers of free energies (or energy-Casimir functionals) under the mass constraints. For the quantum counterpart, that is, the gravitational Hartree equations, isotropic states as free energy minimizers are constructed by Aki, Dolbeault and Sparber. In this talk, we are concerned with the correspondence between quantum and classical isotropic states. Precisely, we prove that as the Planck constant goes to zero, free energy minimizers for the Hartree equation converge to those for the Vlasov equation in terms of potential functions as well as via the Wigner transform/Toplitz quantization.

Homogenization of elliptic and parabolic soft inclusions

Minha Yoo (NIMS)

ABSTRACT

In this talk, we consider periodic Soft inclusion problems of general type elliptic and parabolic non-linear equation of non-divergence form. Usually, it is called Soft inclusion problems to find effective conductivity of composites consisting of a medium with non-conducting grains. Mathematically, non-conducting grains are described by union of disjoint holes with periodicity epsilon. For each epsilon, the unique current density function (the solution of epsilon problem) u-epsilon exists for a given boundary data. We note that, at the boundary of grains, the Neumann data of u-epsilon vanishes. We will present the uniform convergence of u-epsilon and to find ”effective equation” what the limit of u-epsilon satisfy. Moreover, we give an estimate of the rate of convergence of u” to the solution u of homogenized problem.